Quiz 8 Solution

October 16, 2017

1. (2 points) If the **derivative** of $f(x)$ is $f'(x) = 4\ln(x^2 + 47)$, find the x-value of the inflection point(s) of $f(x)$.

   Solution: We have an inflection point if the concavity *changes* at that point; that is, if $f''(x)$ changes from positive to negative or negative to positive.

   First, we find the second derivative:

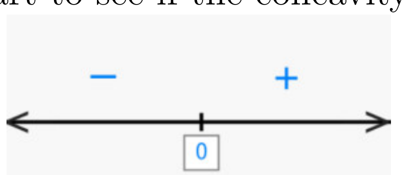
$$\begin{aligned} f''(x) &= \frac{4}{x^2+47} \cdot \frac{d}{dx}[x^2+47] \text{ by Chain Rule} \\ &= \frac{4}{x^2+47} \cdot (2x) \\ &= \frac{8x}{x^2+47} \end{aligned}$$

   Then, we find possible inflection points by setting the numerator and denominator of $f''(x)$ equal to zero:

$$8x = 0 \implies x = 0$$

$$x^2 + 47 = 0 \text{ is never true.}$$

   Finally, we create a sign chart to see if the concavity changes at $x = 0$:



   Since the concavity changes at $x = 0$, we have an inflection point there.

   Answer: $x = 0$

2. (2 points) Find the largest open intervals on which $g(x) = \frac{1}{3}x^3 - x^2 - 8x + 42$ is increasing or decreasing.

   Solution: We take the first derivative and find when it equals zero:

$$\begin{aligned} g'(x) = x^2 - 2x - 8 &= 0 \\ (x-4)(x+2) &= 0 \\ x &= -2,\ x = 4 \end{aligned}$$

Now we create a sign chart to see when the first derivative is positive and negative:

Answer: Decreasing on $(-2, 4)$; increasing on $(\infty, -2) \cup (4, \infty)$

3. (1 point) What did you struggle the most with on Exam 2?
Answer: Answers will vary.